

On-Line Geometric Modeling Notes

BÉZIER PATCH SUBDIVISION

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Overview

A general method can be specified to subdivide a Bézier patch. This method is specified unlike the matrix methods, as it is based upon the definition of the patch as a set of curves..

The Method for Subdivision

We recall that, if we take the analytic equation of a Bézier patch, fix u and group factors appropriately, we obtain

$$\mathbf{P}(u, v) = \sum_{j=0}^m \left[\sum_{i=0}^n \mathbf{P}_{i,j} B_{i,n}(u) \right] B_{j,m}(v)$$

We notice that portion of the equation inside the brackets is the representation of a Bézier curve. If we let $\mathbf{Q}_j(u)$ be the value inside the brackets, i.e.

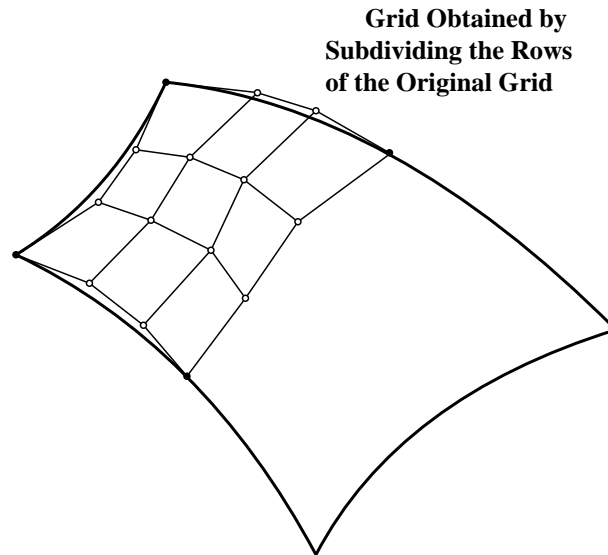
$$\mathbf{Q}_j(u) = \sum_{i=0}^n \mathbf{P}_{i,j} B_{i,n}(u)$$

Then

$$\mathbf{P}(u, v) = \sum_{j=0}^m \mathbf{Q}_j(u) B_{j,m}(v)$$

That is, the quantities $\mathbf{Q}_j(u)$ form the control points of another Bézier curve, and together for all u and v , they form the surface.

If, then, we subdivide each of the m rows of the $P_{i,j}$ matrix, it implies that the Q_j s in the above equation represent only points from the first half of the patch (with respect to u). The following illustration shows the result of subdividing the rows in the 4×4 case.



The second half of the patch can be obtained in a similar fashion. The first and second half of the patch, with respect to v , can be obtained by subdividing the columns.

Summary

So, using only curve methods, and by subdividing the rows or columns of the control point array, we can effectively subdivide a Bézier patch. This is the most frequently used algorithm in software implementations of subdivision and can be utilized for Bézier patches of arbitrary degree.

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