

On-Line Computer Graphics Notes

FRAMES

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Overview

In computer graphics we manipulate objects which may include light sources, cameras, and models in our scenes. Each of these is likely to be defined in its own coordinate system and then placed within the scene we are modeling. We must be able to relate these coordinate systems, both to a global coordinate system and to each other. We place coordinate systems into an affine space through the use of *frames*.

Definition of a Frame

Let \mathcal{A} be an affine space of dimension n . Let \mathbf{O} be a point in this space and let $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ be any basis for \mathcal{A} . We call the collection $\mathcal{F} = (\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n, \mathbf{O})$ a *frame* for \mathcal{A} . Frames form *coordinate systems* in our affine space: If we are given a point \mathbf{P} , then it can be written as $\mathbf{O} + \vec{v}$, where \mathbf{O} is the origin of the frame and \vec{v} is a vector. Since $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ forms a basis for \mathcal{A} , the vector \vec{v} can be written uniquely as

$$\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n$$

and so the point \mathbf{P} can be written uniquely as

$$\mathbf{P} = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n + \mathbf{O}$$

The set of values (c_1, c_2, \dots, c_n) are the *coordinates* of \mathbf{P} relative to the frame \mathcal{F} .

Examples of Frames

Two-dimensional examples of frames are fairly straightforward to produce. These frames contain two vectors and a point representing the origin of the frame. The vectors form a basis for the vector space of all two-dimensional vectors.

- The standard Cartesian frame $(\vec{u}, \vec{v}, \mathbf{O})$ in 2d has $\vec{u} = \langle 1, 0 \rangle$, $\vec{v} = \langle 0, 1 \rangle$ and $\mathbf{O} = (0, 0)$. Most readers will equate the coordinates (x, y) to points in this frame given by

$$x\vec{u} + y\vec{v} + \mathbf{O}$$

(We can define a Cartesian frame for any dimension. It consists of the origin $(0, 0, \dots, 0)$, and the vectors $\langle 1, 0, \dots, 0 \rangle$, $\langle 0, 1, \dots, 0 \rangle$, ..., $\langle 0, 0, \dots, 1 \rangle$.)

- Consider the frame given by the origin $(0, 0)$ and the two vectors $\vec{u} = (1, 0)$ and $\vec{v} = (1, 1)$. The point \mathbf{P} , having coordinates $(5, 3)$ in this frame, can be written as

$$5\vec{u} + 3\vec{v} + \mathbf{O}$$

or

$$5 \langle 1, 0 \rangle + 3 \langle 1, 1 \rangle + (0, 0)$$

which is the Cartesian coordinate $(8, 3)$.

- Consider the frame given by the origin $(2, 2)$ and the two vectors $\vec{u} = (1, 0)$ and $\vec{v} = (0, 2)$. The point \mathbf{P} , having coordinates $(5, 3)$ in this frame, can be written as

$$5\vec{u} + 3\vec{v} + \mathbf{O}$$

or

$$5 \langle 1, 0 \rangle + 3 \langle 0, 2 \rangle + (2, 2)$$

which is the Cartesian coordinate $(7, 8)$.

Matrix representation of Points and Vectors

Points and vectors can be uniquely identified by the coordinates relative to a specific frame. Given a frame $(\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n, \mathbf{O})$ in an affine space \mathcal{A} , we can write a point \mathbf{P} uniquely as

$$\mathbf{P} = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n + \mathbf{O}$$

This can also be written in matrix notation as

$$\mathbf{P} = \begin{bmatrix} c_1 & c_2 & \dots & c_n & 1 \end{bmatrix} \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vdots \\ \vec{v}_n \\ \mathbf{O} \end{bmatrix}$$

Similarly, since the vectors of our affine space form a vector space, we can write a vector \vec{v} uniquely as

$$\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n$$

which can be written as

$$\mathbf{P} = \begin{bmatrix} c_1 & c_2 & \dots & c_n & 0 \end{bmatrix} \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vdots \\ \vec{v}_n \\ \mathbf{O} \end{bmatrix}$$

Thus, points are represented as row vectors whose last component is 1 and vectors are represented as row vectors whose last component is 0 (zero).

References

- [1] DEROSE, T. Coordinate-free geometric programming. Technical Report 89-09-16, Department of Computer Science and Engineering, University of Washington, Seattle, Washington, 1994.
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