

On-Line Computer Graphics Notes

HOMOGENEOUS COORINDATE SYSTEMS

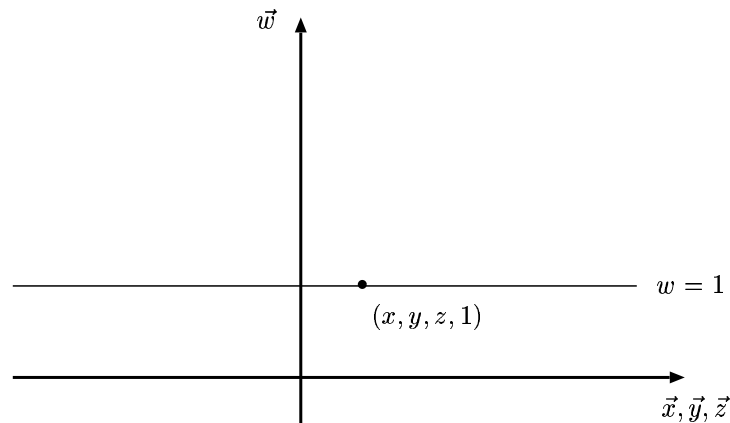
Kenneth I. Joy
Visualization and Graphics Research Group
Department of Computer Science
University of California, Davis

A 4-dimensional homogeneous coordinate system is frequently useful in representing the projective operations inherent in computer graphics systems. It has two principal uses:

- it allows the use of 4×4 matrices to represent general 3-dimensional transformations
- it allows a simplified representation of mathematical functions – the rational form – in which rational polynomial functions can be simply represented.

We will concentrate on the first of these in these notes.

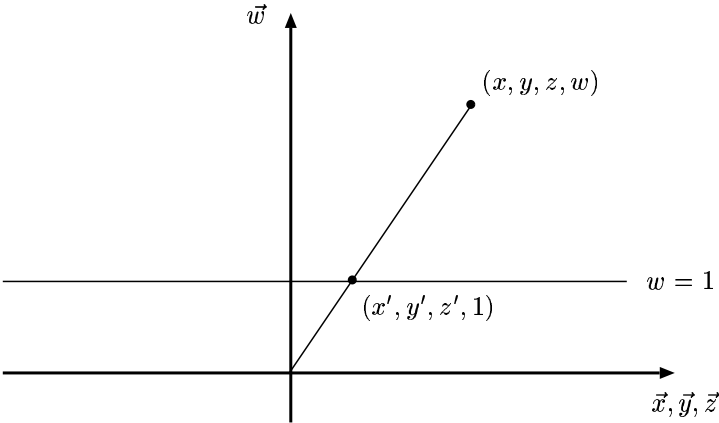
Points in four-dimensional homogeneous space are usually specified by (x, y, z, w) . This is an extension of the usual coordinate representation that we utilize with frames $(x, y, z, 1)$. If we look at the coordinates $(x, y, z, 1)$ in four dimensions, we notice that they all lie on the space $w = 1$. This is illustrated in the figure below.



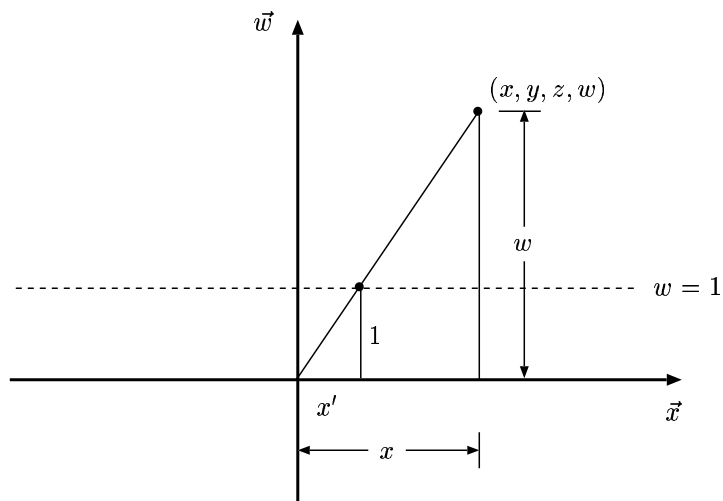
The transformations that we have defined all transform the fourth coordinate to one, thereby limiting the movement of the points to the $w = 1$ space. However, it will be necessary to utilize transformations that move the points away from this space creating a conflict – as long as the points are in the $w = 1$ space we can consider them as being in an exact copy of three-dimensional space ; if they move away from this space, how do we identify them with three-dimensional points any longer?

Well there are many ways to get a point away from the $w = 1$ space to identify with a point in the space. We will utilize a *projective* approach in which we will identify points out of the $w = 1$ space with points in the space as follows:

Given a coordinate (x, y, z, w) , we identify the point $(x', y', z', 1)$ in the $w = 1$ space with (x, y, z, w) if the point $(x', y', z', 1)$ is the unique point on the line connecting (x, y, z, w) with the origin.



It is fairly easy to calculate the simple formula for identification of these points. If we look at the following figure, where we have focussed on two axes (the w and the x axis),



we can utilize a similar triangle argument to obtain that

$$x' = \frac{x}{w}$$

and by considering projections onto the other axes, we can also obtain that

$$y' = \frac{y}{w}$$

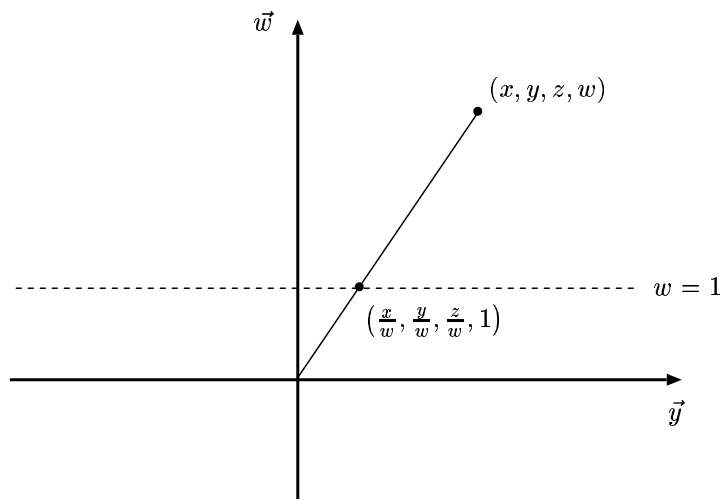
and

$$z' = \frac{z}{w}$$

Thus we have the identification

$$(x, y, z, w) \text{ identifies with } \left[\frac{x}{w}, \frac{y}{w}, \frac{z}{w}, 1 \right]$$

which is shown in the following figure



This mapping has the following properties:

- The space $w = 0$ is not in the domain of this mapping. Clearly if we consider the point $(x, y, z, 0)$, the line through this point and the origin does not intersect the space $w = 1$.
- The mapping is many-to-one, in that any point (xw, yw, zw, w) also maps to the point (x, y, z) for $w \neq 0$.
- Points (x, y, z, w) with $w < 0$ are in the domain of this mapping, as the line through this point and the origin does intersect the space $w = 1$.

The correspondence between 3-dimensional space and 4-dimensional homogeneous space is frequently utilized to apply 4-dimensional functions that define operators on 3-dimensional points. If we are given a coordinate (x, y, z) in 3-dimensional space, we apply a 4×4 matrix to the coordinate by multiplying

$$\begin{bmatrix} x & y & z & 1 \end{bmatrix} \begin{bmatrix} 4 \times 4 \end{bmatrix} = \begin{bmatrix} x' & y' & z' & w \end{bmatrix}$$

The point (x', y', z', w') in homogeneous space is then projects to the space $w = 1$ by dividing by the fourth coordinate

$$\left[\frac{x'}{w}, \frac{y'}{w}, \frac{z'}{w}, 1 \right]$$

and is then identified with the coordinate $(\frac{x'}{w}, \frac{y'}{w}, \frac{z'}{w})$ in 3-dimensional space, which is considered to be the result of the transformation.

**All contents copyright (c) 1996, 1997, 1998, 1999
Computer Science Department, University of California, Davis
All rights reserved.**