On-Line Computer Graphics Notes

SHEARING

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Overview

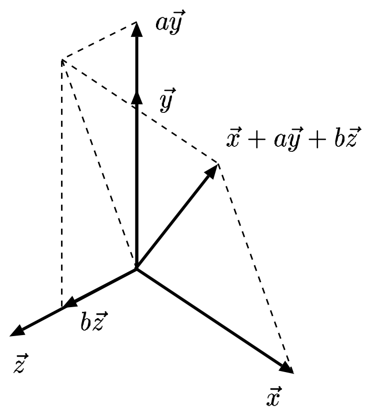
Shearing transformations in three-dimensions alter two of the three coordinate values proportionally to the value of the third coordinate.

The X-Shear Transformation

Given a frame $\mathcal{F} = (\vec{u}, \vec{v}, \vec{w}, \mathbf{O})$. We "x-shear" a frame by modifying the first vector of the frame by adding to it a linear combination of the other two vectors. The frame transformation takes the following form

$$\begin{bmatrix} \vec{u} \\ \vec{v} \\ \vec{w} \\ \mathbf{O} \end{bmatrix} \longrightarrow \begin{bmatrix} \vec{u} + a\vec{v} + b\vec{w} \\ \vec{v} \\ \vec{w} \\ \mathbf{O} \end{bmatrix}$$

An illustration of this process is given in the following figure.



This transform can be implemented by the following 4×4 matrix:

$$\begin{bmatrix} 1 & a & b & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \vec{u} \\ \vec{v} \\ \vec{w} \\ \mathbf{O} \end{bmatrix} = \begin{bmatrix} \vec{u} + a\vec{v} + b\vec{w} \\ \vec{v} \\ \vec{w} \\ \mathbf{O} \end{bmatrix}$$

and so we define the x-shear transformation by

$$H_{x;a,b} = \begin{bmatrix} 1 & a & b & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If this transformation is applied to the point (u, v, w), we obtain

$$\begin{bmatrix} u & v & w & 1 \end{bmatrix} \begin{bmatrix} 1 & a & b & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u & au + v & bu + w & 1 \end{bmatrix}$$

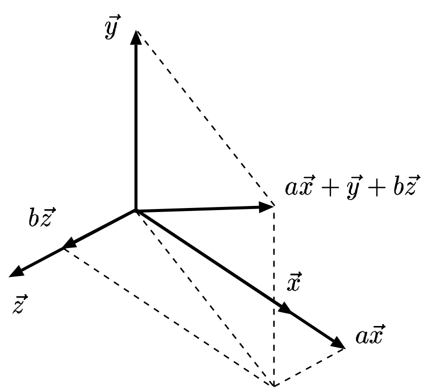
and thus objects can be sheared by applying this matrix to all points of the object.

The Y-Shear Transformation

Given a frame $\mathcal{F} = (\vec{u}, \vec{v}, \vec{w}, \mathbf{O})$. We "y-shear" a frame by transforming the second vector by adding a linear combination of the other two vectors. The frame transformation takes the following form

$$\begin{bmatrix} \vec{u} \\ \vec{v} \\ \vec{w} \\ \mathbf{O} \end{bmatrix} \longrightarrow \begin{bmatrix} \vec{u} \\ a\vec{u} + \vec{v} + b\vec{w} \\ \vec{w} \\ \mathbf{O} \end{bmatrix}$$

An illustration of this process is given in the following figure.



This transform can be implemented by the following 4×4 matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \vec{u} \\ \vec{v} \\ \vec{w} \\ \mathbf{O} \end{bmatrix} = \begin{bmatrix} \vec{u} \\ a\vec{u} + \vec{v} + b\vec{w} \\ \vec{w} \\ \mathbf{O} \end{bmatrix}$$

and so we define the y-shear transformation by

$$H_{y;a,b} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If this transformation is applied to the point (u, v, w), we obtain

$$\begin{bmatrix} u & v & w & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u + av & v & w + bv & 1 \end{bmatrix}$$

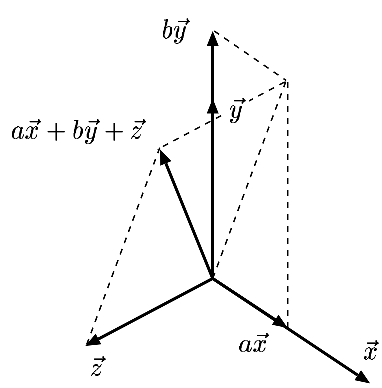
and thus objects can be sheared by applying this matrix to all points of the object.

The Z-Shear Transformation

Given a frame $\mathcal{F} = (\vec{u}, \vec{v}, \vec{w}, \mathbf{O})$. We "z-shear" a frame by transforming the third vector by adding a linear combination of the other two vectors. The frame transformation takes the following form

$$\begin{bmatrix} \vec{u} \\ \vec{v} \\ \vec{w} \\ \mathbf{O} \end{bmatrix} \longrightarrow \begin{bmatrix} \vec{u} \\ \vec{v} \\ a\vec{u} + b\vec{v} + \vec{w} \\ \mathbf{O} \end{bmatrix}$$

which is illustrated by



This transform can be implemented by the following 4×4 matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ a & b & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \vec{u} \\ \vec{v} \\ \vec{w} \\ \mathbf{O} \end{bmatrix} = \begin{bmatrix} \vec{u} \\ \vec{v} \\ a\vec{u} + b\vec{v} + \vec{w} \\ \mathbf{O} \end{bmatrix}$$

and so we define the z-shear transformation by

$$H_{z;a,b} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ a & b & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If this transformation is applied to the point (u, v, w), we obtain

$$\begin{bmatrix} u & v & w & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ a & b & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u + aw & v + bw & w & 1 \end{bmatrix}$$

and thus objects can be sheared by applying this matrix to all points of the object.
